Microelectronics
Circuit Analysis and Design

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Chapter 6

Basic BJT Amplifiers
In this chapter, we will:

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.
- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-emitter amplifier.
    - Understand the ac load line & determine the maximum symmetrical swing of the output.
  - Analyze the emitter-follower amplifier.
  - Analyze the common-base amplifier.
- Analyze multitransistor or multistage amplifiers.
- Understand the concept of signal power gain in an amplifier circuit.
Common Emitter with Time-Varying Input
I_B Versus V_{BE} Characteristic

\[ i_B \approx I_{BQ} \left(1 + \frac{V_{be}}{V_T}\right) = I_B + i_b \]
ac Equivalent Circuit for Common Emitter
Small-Signal Hybrid $\pi$ Model for npn BJT

Phasor signals are shown in parentheses.

$g_m = \frac{I_{CQ}}{V_T}$

$r_\pi = \frac{\beta V_T}{I_{CQ}}$

$g_m r_\pi = \beta$
Small-Signal Equivalent Circuit Using Common-Emitter Current Gain

\[ i_b (I_b) \quad i_c (I_c) \]

\[ v_{be} (V_{be}) \quad \beta i_b (\beta I_b) \quad v_{ce} (V_{ce}) \]

\[ r_\pi \quad i_e (I_e) \]
Small-Signal Equivalent Circuit for npn Common Emitter circuit

\[ A_v = -\left( g_m R_C \right) \left( \frac{r_\pi}{r_\pi + R_B} \right) \]
Problem-Solving Technique: BJT AC Analysis

1. Analyze circuit with only dc sources to find Q point.

2. Replace each element in circuit with small-signal model, including the hybrid π model for the transistor.

3. Analyze the small-signal equivalent circuit after setting dc source components to zero.
## Transformation of Elements

<table>
<thead>
<tr>
<th>Element</th>
<th>DC Model</th>
<th>AC Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>$R$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>Open</td>
<td>$C$</td>
</tr>
<tr>
<td>Inductor</td>
<td>Short</td>
<td>$L$</td>
</tr>
<tr>
<td>Diode</td>
<td>$+V_g$, $r_f$ - $r_d = V_T/I_D$</td>
<td></td>
</tr>
<tr>
<td>Independent Constant Voltage Source</td>
<td>$+ V_S$ -</td>
<td>Short</td>
</tr>
<tr>
<td>Independent Constant Current Source</td>
<td>$I_S$</td>
<td>Open</td>
</tr>
</tbody>
</table>
Hybrid $\pi$ Model for npn with Early Effect

\[ r_o = \frac{V_A}{I_{CQ}} \]
Hybrid p Model for pnp with Early Effect

(a) Hybrid p Model for pnp with Early Effect

(b) Hybrid p Model for pnp with Early Effect

\[ I_b, I_c, V_{eb}, V_{pi}, r_{pi}, g_m V_{pi}, r_o, V_{ec} \]

\[ \beta I_b, r_o, V_{ec} \]
Single Transistor Amplifier Analysis: Summary of Procedure

Steps to Analyze a Transistor Amplifier

1.) Determine DC operating point and calculate small signal parameters (see next page)

2.) Convert to the AC only model.
   • DC Voltage sources are shorts to ground
   • DC Current sources are open circuits
   • Large capacitors are short circuits
   • Large inductors are open circuits

3.) Use a Thevenin circuit (sometimes a Norton) where necessary. Ideally the base should be a single resistor + a single source. **Do not confuse this with the DC Thevenin you did in step 1.**

4.) Replace transistor with small signal model

5.) Simplify the circuit as much as necessary.

6.) Calculate the small signal parameters ($r_{\pi}$, $g_m$, $r_o$ etc...) and then gains etc…
Single Transistor Amplifier Analysis

**Step 1 detail**

**DC Bias Point**

\[ 3V = I_E R_E + V_{be} + I_B R_{TH} \]

\[ 3V = I_C \left( \frac{\beta_o + 1}{\beta_o} \right) R_e + 0.7V + I_B R_{TH} \]

\[ 3V = I_B \beta_o \left( \frac{\beta_o + 1}{\beta_o} \right) R_e + 0.7V + I_B R_{TH} \]

\[ 3V = I_B (100+1) 1300 + 0.7 + I_B 7500 \]

\[ I_B = 16.6 \text{ uA}, \quad I_C = I_B \beta_o = 1.66 \text{ mA}, \quad I_E = (\beta_o + 1) \frac{I_C}{\beta_o} = 1.67 \text{ mA} \]
Single Transistor Amplifier Analysis

Step 6 detail  Calculate small signal parameters

Transconductance  \( g_m = \frac{I_C}{V_T} \approx 40 \frac{I_C}{0.0664} = 0.0664 \text{ S} \)

Input Resistance  \( r_\pi = \frac{\beta_o V_T}{I_C} = \frac{\beta_o}{g_m} = 1506 \text{ } \Omega \)

Output Resistance  \( r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} = 45.2 \text{ K } \Omega \)

\[ V_{TH} = 0.88 \text{ } V_s \]

\[ r_\pi = 880 \text{ } \Omega \]

\[ V_{be} \]

\[ I_\pi \]

\[ g_m \text{ } V_{be} \]

\[ V_{out} \]

\[ R_L = R_C \mid R_3 \mid r_o \]

\[ v_{out} = -g_m v_{be} R_L \text{ and } v_{be} = v_{Th} \frac{r_\pi}{R_{Th} + r_\pi} \text{ and } v_{Th} = 0.88 v_S \]

\[ A_v \equiv \text{Voltage Gain} = \frac{v_{out}}{v_S} = \left( \frac{v_{out}}{v_{be}} \right) \left( \frac{v_{be}}{v_{th}} \right) \left( \frac{v_{th}}{v_S} \right) = (- g_m R_L) \left( \frac{r_\pi}{R_{Th} + r_\pi} \right)(0.88) \]

\[ A_v = (-0.0664)(45,200 \parallel 4300 \parallel 100,000) \left( \frac{1506}{880 + 1506} \right)(0.88) \]

\[ A_v = -139 \text{ } V/V \]
Detailed Example: Single Transistor Amplifier Analysis
Step 1: Determine DC Operating Point
Remove the Capacitors

Because the impedance of a capacitor is $Z = 1/(j\omega C)$, capacitors have infinite impedance or are open circuits in DC ($\omega = 0$).

Inductors (not present in this circuit) have an impedance $Z = j\omega L$, and are shorts in DC.
Step 1: Determine DC Operating Point
Determine the DC Thevenin Equivalent

Replace all connections to the transistor with their Thevenin equivalents.
Step 1: Determine DC Operating Point
Calculate Small Signal Parameters

Identify the type of transistor (npn in this example) and draw the base, collector, and emitter currents in their proper direction and their corresponding voltage polarities.

Applying KVL to the controlling loop (loop 1):

\[ V_{THB} - I_B R_{THB} - V_{BE} - I_E R_E = 0 \]

Applying KCL to the transistor:

\[ I_E = I_B + I_C \]

Because \( I_C = \beta I_B \),

\[ I_E = I_B + I_C = I_B + \beta I_B = I_B (1 + \beta) \]

Substituting for \( I_E \) in the loop equation:

\[ V_{THB} - I_B R_{THB} - V_{BE} - I_B (1 + \beta) R_E = 0 \]
Step 1: Determine DC Operating Point
Plug in the Numbers

\[ V_{THB} - I_B R_{THB} - V_{BE} - I_B (1+\beta) R_E = 0 \]
\[ V_{THB} - V_{BE} - I_B (R_{THB} + (1+\beta) R_E) = 0 \]
\[ V_{THB} = 12 R_1 / (R_1 + R_2) = 3 \text{ V} \]
\[ R_{THB} = R_1 \parallel R_2 = 7.5 \text{ kΩ} \]
Assume \( V_{BE} = 0.7 \text{ V} \)
Assume \( \beta \) for this particular transistor is given to be 100.

\[ 3 - 0.7 - I_B (7500 + (1+100) \times 1300) = 0 \]
\[ I_B = 16.6 \text{ μA} \]
\[ I_C = \beta I_B = 1.66 \text{ mA} \]
\[ I_E = I_B + I_C = 1.676 \text{ mA} \]
Step 2: Convert to AC-Only Model
Short the Capacitors and DC Current Sources

- DC voltage sources are shorts (no voltage drop/gain through a short circuit).
- DC current sources are open (no current flow through an open circuit).
- Large capacitors are shorts (if \( C \) is large, \( 1/j\omega C \) is small).
- Large inductors are open (if \( L \) is large, \( j\omega L \) is large).
Step 2: Convert to AC-Only Model (Optional) Simplify Before Thevenizing
Step 3: Thevenize the AC-Only Model

\[ v_{thB} = v_s \times \frac{(r_1||r_2)}{(r_2 + [r_1||r_2])} \]

\[ r_{thB} = r_s||r_1||r_2 \]

\[ r_{thC} = r_c||r_L \]

\[ r_{thE} = 0 \Omega \]

\[ v_{thE} = 0 \text{ V} \]

\[ v_{thC} = 0 \text{ V} \]
Step 4: Replace Transistor With Small Signal Model

After replacing the transistor, apply Ohm’s Law: \( V = IR \) to find \( v_{\text{out}} \).

\[ r_{\text{thC}} = r_c||r_L \]

\[ r_{\text{thE}} = 0 \Omega \]

\[ v_{\text{thE}} = 0 \text{ V} \]

\[ v_{\text{thB}} = V_s \ast \left[ (r_1||r_2)/(r_s+r_1||r_2) \right] \]

\[ r_{\text{thB}} = r_s||r_1||r_2 \]

\[ 1.58 \text{ k}\Omega \]

\[ 4.12 \text{ k}\Omega \]

\[ v_{\text{thC}} = 0 \text{ V} \]

\( r_o \) and \( r_{\text{thC}} \) are in parallel, so that Ohm’s Law becomes:

\[ v_{\text{out}} = -IR = -(g_{m}v_{\text{BE}})(r_o||r_{\text{thC}}) \]

Because \( r_{\text{thC}} = r_c||r_L \),

\[ v_{\text{out}} = -(g_{m}v_{\text{BE}})(r_o||r_c||r_L) \]

\[ v_{\text{out}}/v_{\text{BE}} = -g_m(r_o||r_c||r_L) \text{ is the gain from transistor input (v_{BE}) to transistor/circuit output v_{out}} \]
Step 5: Calculate Gain and Small Signal Parameters

As previously determined:
\[ \frac{v_{thB}}{v_s} = \frac{r_1||r_2}{(r_1||r_2) + r_2} \]

Applying a voltage divider:
\[ \frac{v_{BE}}{v_{thB}} = \frac{r_x}{r_x + r_{thB}} \]

Gain factor:
\[ \frac{v_{out}}{v_{BE}} = -g_m(r_0||r_c||r_L) \]

Because calculating the DC operating point was done first, we have equations for \( g_m \), \( r_\pi \), and \( r_o \) in terms of previously calculated DC currents and voltages.

Transconductance \( g_m = \frac{I_C}{V_T} \approx 40I_C \)

Input Resistance \( r_\pi = \frac{\beta_0V_T}{I_C} = \frac{\beta_0}{g_m} \)

Output Resistance \( r_o = \frac{V_A + V_{CE}}{I_C} \)

Plugging in the numbers:
Gain = \( \frac{v_{out}}{v_s} = -139 \text{ V/V} \)
Interpretation/Analysis of Results

Gain = \frac{v_{out}}{v_s} = \left(\frac{v_{thB}}{v_s}\right)\left(\frac{v_{BE}}{v_{thB}}\right)\left(\frac{v_{out}}{v_{BE}}\right) = -139 \text{ V/V}

Both terms are loss factors, i.e. they can never be greater than 1 in magnitude and thus cause the gain to decrease.

This term is the gain factor and is responsible for amplifying the signal.

\begin{align*}
\frac{v_{thB}}{v_s} &= \frac{(r_1||r_2)}{([r_1||r_2] + r_s)} \\
v_{BE}/v_{thB} &= \frac{r_x}{(r_x + r_{thB})} \\
v_{out}/v_{BE} &= -g_m(r_o||r_c||r_L)
\end{align*}

The AC input signal has been amplified 139 times in magnitude. The negative sign indicates there has been a phase shift of 180°.
Expanded Hybrid $\pi$ Model for npn
h-Parameter Model for npn

\[ h_{ie} = r_b + r_\pi || r_\mu \]

\[ h_{fe} = \beta \]

\[ h_{re} \approx \frac{r_\pi}{r_\mu} \]

\[ h_{oe} = \frac{1 + \beta}{r_\mu} + \frac{1}{r_o} \]
T-Model of an npn BJT

\[ g_m v_{be} = \alpha i_e \]

\[ r_e = \frac{V_T}{I_E} \]
4 Equivalent 2-port Networks

Voltage Amplifier

\[ V_{in} \rightarrow R_i \rightarrow A_{vo} V_{in} \rightarrow V_o \]

Current Amplifier

\[ i_{in} \rightarrow R_i \rightarrow A_{is} i_{in} \rightarrow i_o \]
4 Equivalent 2-port Networks

Transconductance Amplifier

Transresistance Amplifier
Common Emitter with Voltage-Divider Bias and a Coupling Capacitor

\[ V_{CC} = 12 \text{ V} \]

\[ R_1 = 93.7 \text{ k}\Omega \]

\[ R_C = 6 \text{ k}\Omega \]

\[ R_S = 0.5 \text{ k}\Omega \]

\[ R_2 = 6.3 \text{ k}\Omega \]
Small-Signal Equivalent Circuit – Coupling Capacitor Assumed a Short
npn Common Emitter with Emitter Resistor

\[ V_{CC} = 10 \, \text{V} \]

\[ R_S = 0.5 \, \text{k}\Omega \]
\[ R_1 = 56 \, \text{k}\Omega \]
\[ R_C = 2 \, \text{k}\Omega \]
\[ R_2 = 12.2 \, \text{k}\Omega \]
\[ R_E = 0.4 \, \text{k}\Omega \]
Small-Signal Equivalent Circuit: Common Emitter with $R_E$

$$R_{ib} = r_\pi + (1 + \beta)R_E$$

$$R_i = R_1 \parallel R_2 \parallel R_{ib}$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} \left( \frac{R_i}{R_i + R_S} \right)$$
\( R_E \) and Emitter Bypass Capacitor

\[ V^+ = +5 \text{ V} \]
\[ V^- = -5 \text{ V} \]

\[ R_S = 0.5 \text{ k}\Omega \]
\[ R_B = 100 \text{ k}\Omega \]

\[ R_C \]
\[ v_O \]
\[ v_s \]
Problem-Solving Technique: Maximum Symmetrical Swing

1. Write dc load line equation that relates \( I_{CQ} \) and \( V_{CEQ} \).
2. Write ac load line equations that relates \( i_c \) and \( v_{ce} \).
3. In general, \( i_c = I_{CQ} - I_C(\text{min}) \), where \( I_C(\text{min}) \) is zero or other minimum collector current.
4. In general, \( v_{ce} = V_{CEQ} - V_{CE}(\text{min}) \), where \( V_{CE}(\text{min}) \) is some specified minimum collector-emitter voltage.
5. Combine above 4 equations to find optimum \( I_{CQ} \) and \( V_{CEQ} \).
Common-Collector
or Emitter-Follower Amplifier

\[ V_{CC} = 5 \text{ V} \]

\[ R_S = 0.5 \text{ k}\Omega \quad C_C \quad R_1 = 50 \text{ k}\Omega \]

\[ R_2 = 50 \text{ k}\Omega \quad R_E = 2 \text{ k}\Omega \]

\[ v_s \quad \text{ and } \quad v_o \]
Small-Signal Equivalent Circuit: Emitter Follower

\[ R_{ib} = r_\pi + (1 + \beta)(r_o \parallel R_E) \]

\[ R_i = R_1 \parallel R_2 \parallel R_{ib} \]

\[ A_v = \frac{(1 + \beta)(r_o \parallel R_E)}{r_\pi + (1 + \beta)(r_o \parallel R_E)} \left( \frac{R_i}{R_i + R_S} \right) \]
Output Resistance: Emitter Follower

\[ R_o = \frac{r_r}{1 + \beta} || R_E || r_o \]
Common-Base Amplifier
Small-Signal Equivalent Circuit: Common Base

\[ \begin{align*}
A_v &= g_m (R_C \| R_L) \\
A_i &= g_m \left( \frac{R_C}{R_C + R_L} \right) \left[ \frac{r_\pi}{1 + \beta} \right] R_E
\end{align*} \]
Input Resistance: Common Base

\[ R_{ie} = \frac{r_{\pi}}{1+\beta} \]
Output Resistance: Common Base

\[ R_O = R_C \]
Common Emitter Cascade Amplifier

\[ v_s \]

\[ V^+ = +5 \text{ V} \]

\[ V^- = -5 \text{ V} \]

\[ R_S = 0.5 \text{ k}\Omega \]

\[ R_1 = 100 \text{ k}\Omega \]

\[ R_{C1} = 5 \text{ k}\Omega \]

\[ R_{E1} = 2 \text{ k}\Omega \]

\[ R_2 = 50 \text{ k}\Omega \]

\[ R_{E2} = 2 \text{ k}\Omega \]

\[ R_{C2} = 1.5 \text{ k}\Omega \]

\[ R_L = 5 \text{ k}\Omega \]

\[ Q_1 \]

\[ Q_2 \]

\[ C_{C1} \]

\[ C_{E1} \]

\[ C_{E2} \]

\[ C_{C2} \]

\[ v_o \]
Small-Signal Equivalent Circuit: Cascade Amplifier
Darlington Pair

\[ A_i \approx \beta_1 \beta_2 \]
Cascode Amplifier

(a)

(b)
Small-Signal Equivalent Circuit: Cascode Amplifier

\[ A_v \approx -g_{m1} \left( R_C \parallel R_L \right) \]