CHAPTER 3
FORCES & NEWTON’S LAW OF MOTION

3.1 The concepts of force and mass

FORCE

❖ A force is a push or pull upon an object resulting from the object's interaction with another object.
❖ Force is a quantity which is measured using the standard metric unit known as the Newton, N.

\[ 1 \text{ Newton} = 1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \]

❖ Force is a vector quantity.
❖ There must be a net force (unbalanced force) acting on an object for the object to change its velocity.
❖ Net force, \( \Sigma F \), is the vector sum or the unbalanced force acting on an object.
❖ Force can be divided into:

a. Contact forces—physical contact between objects—frictional forces, tensional forces, normal forces
b. Non-contact forces—action-at-a-distance forces—two objects are not in physical contact with each other—gravitational forces

Examples of action-at-a-distance forces include gravitational forces (e.g., the sun and planets exert a gravitational pull on each other despite their large spatial separation; even when your feet leave the earth and you are no longer in contact with the earth, there is a gravitational pull between you and the Earth), electric forces (e.g., the protons in the nucleus of an atom and the electrons outside the nucleus experience an electrical pull towards each other despite their small spatial separation), and magnetic forces (e.g., two magnets can exert a magnetic pull on each other even when separated by a distance of a few centimeters.)
- A measure of an object’s inertia.
- Quantity of matter
- The difference between mass & weight are show below:

<table>
<thead>
<tr>
<th>Mass</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit: kg</td>
<td>Unit: Newton</td>
</tr>
<tr>
<td>Not referred to gravity</td>
<td>Always referred to gravity</td>
</tr>
</tbody>
</table>

( Giancoli 6th Ed. Pg 75)

**EXAMPLE 3.1:** A book on a horizontal desk.

There are two forces acting on the book:

i. Gravity pulls downwards so the weight, \( W \) is always present, \( W = mg \)
ii. Table pushes upwards and a force result is normal force, \( N \) which is perpendicular to the contact surface.

### 3.2 Newton’s Laws of Motion

**Newton’s first law**

If no forces act on an object, it continues in its original state of motion; that is, unless something exerts an external force on it, an object at rest remains at rest and an object moving with some velocity continues with that same velocity.

Newton’s first law also known as the Law of Inertia. Inertia is a property of a body to make the body to remain in its state, either stationary or moving with constant velocity.

*Mass* is a measure of inertia. It is a physical quantity that determines how difficult it is to accelerate or decelerate an object. SI unit is kg.

**Newton’s second law**

When a net external force \( \Sigma F \) acts on an object of mass \( m \), the acceleration \( a \) that results is directly proportional to the net force and inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

\[
a = \frac{\Sigma F}{m} \text{ or } \Sigma F = ma
\]

SI unit of force: kg.m/s\(^2\) = Newton, N
**Newton’s third law:**
When two bodies interact, they exert equal but opposite forces on each other. Example: Forces come in pairs, if a hammer exerts a force on a nail; the nail exerts an equal but oppositely directed force on the hammer.

*i.e.* \( F_{AB} = -F_{BA} \)

Action force = Reaction force

Note that they act on different bodies, so they will not cancel.

### 3.3 Types of forces

**The Gravitational Force**
The gravitational force is a mutual force of attraction between any two objects. Expressed by Newton’s Law of Universal Gravitation:

For two particles that have masses \( m_1 \) and \( m_2 \) are separated by a distance \( r \), the force that each exerts on the other is directed along the line joining the particles and has a magnitude given by

\[
F = G \frac{m_1 m_2}{r^2}
\]

where \( G \) is the universal gravitational constant

\[
G = 6.673 \times 10^{-11} \text{ Nm}^2/\text{kg}^2
\]

The weight of an object on or above the earth is the gravitational force that the earth exerts on the object:

\[
W = G \frac{M_E m}{r^2}
\]

where \( M_E \) is the mass of the earth

Gravity Near the Earth’s Surface

\[
M_E g = W = G \frac{M_E m}{r^2}
\]

**The Normal Force**
The normal force \( N \), is the force that a surface exerts on an object with which it is in contact. The normal force is exerted perpendicular to the surface.

**The Tension Force**
Tension \( T \), is the force which is transmitted through a string, rope, or wire when it is pulled tight by forces acting at each end. The tensional force is directed along the wire and pulls equally on the objects on either end of the wire.
The Frictional Force
When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion. The resistance is called the force of friction. The friction force is the force exerted by a surface as an object moves across it or makes an effort to move across it. The friction force opposes the motion of the object. For example, if a book moves across the surface of a desk, the desk exerts a friction force in the direction.

**Static Friction, }**

Static friction acts to keep the object from moving. The magnitude of the static friction depends on the magnitude of the applied force and can have the values up to a maximum of

\[ f_s = u_s N \]

where \( u_s \) = the coefficient of static friction

**Kinetic Friction, }**

The force of kinetic friction acts when the object is in motion. The magnitude given by

\[ f_k = u_k N \]

where \( u_k \) = the coefficient of kinetic friction

**EXAMPLE 3.2**

A force of 48.0N is required to start a 5.0kg box moving across a horizontal concrete floor. A) What is the coefficient of static friction between the box and the floor? B) If the 48.0N force continues, the box accelerates at 0.7 m/s\(^2\). What is the coefficient of kinetic friction?

**Solutions:**

A free-body diagram for the box is shown. Since the box does not accelerate vertically, \( F_N = mg \)

\[ (a) \] To start the box moving, the pulling force must just overcome the force of static friction, and that means the force of static friction will reach its maximum value of \( F_u = \mu_s F_N \). Thus we have for the starting motion,

\[ F_p = F_u = \mu_s F_N = \mu_s mg \rightarrow \mu_s = \frac{F_p}{mg} = \frac{48.0 \text{ N}}{5.0 \text{ kg} \cdot (9.8 \text{ m/s}^2)} = 0.98 \]

\[ (b) \] The same force diagram applies, but now the friction is kinetic friction, and the pulling force is NOT equal to the frictional force, since the box is accelerating to the right.

\[ \sum F = F_p - F_u = ma \rightarrow F_p - \mu_k F_N = ma \rightarrow F_p - \mu_k mg = ma \rightarrow \]

\[ \mu_k = \frac{F_p - ma}{mg} = \frac{48.0 \text{ N} - (5.0 \text{ kg})(0.70 \text{ m/s}^2)}{(5.0 \text{ kg})(9.8 \text{ m/s}^2)} = 0.91 \]
EXAMPLE 3.5:
Calculate the acceleration due to gravity on the Moon. The Moon’s radius is $1.74 \times 10^6$ m and its mass is $7.35 \times 10^{22}$ kg.

Solutions:
The force of gravity on an object at the surface of a planet is given by Newton’s law of Universal Gravitation, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely-falling object is acceleration due to gravity.

\[
F_g = G \frac{M_{\text{Moon}} m}{r_{\text{Moon}}^2} = m g_{\text{Moon}} \rightarrow
\]

\[
g_{\text{Moon}} = G \frac{M_{\text{Moon}}}{r_{\text{Moon}}^2} = \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \left(\frac{7.35 \times 10^{22} \text{ kg}}{1.74 \times 10^6 \text{ m}}\right)^2 = 1.62 \text{ m/s}^2
\]

EXAMPLE 3.6:
Two blocks are in contact on a frictionless table. A horizontal force is applied to one block, as shown in figure.

(a) If $m_1 = 2.3 \text{ kg}$, $m_2 = 1.2 \text{ kg}$, and $F = 3.2 \text{ N}$, find the force of contact between the two blocks.

(b) Show that if the same force $F$ is applied to $m_2$, but not to $m_1$, the force of contact between the blocks is $2.1 \text{ N}$, which is not the same value derived in (a). Explain.

Answer:

\[
F \rightarrow m_1 \quad \text{and} \quad N \rightarrow m_2
\]

a) Consider mass $m_1$
\[
F - N = m_1 a \quad \text{(1)}
\]

Consider mass $m_2$
\[
N = m_2 a \quad \text{(2)}
\]

(1) + (2) gives
\[
F = (m_1 + m_2) a
\]
\[
a = \frac{F}{m_1 + m_2}
\]
\[
\therefore N = \frac{m_2 F}{m_1 + m_2} \quad \text{(3)}
\]
for $m_1 = 2.3 \text{ kg}$, $m_2 = 1.2 \text{ kg}$, and $F = 3.2 \text{ N}$,
$N = 1.10 \text{ N}$

b) Interchange $m_1$ and $m_2$, one obtains
$N = 2.1 \text{ N}$
In order to preserve constant acceleration, greater force of the contact is needed to move the mass $m_1$.

**EXAMPLE 3.7:**
The two blocks, $m = 16 \text{ kg}$ and $M = 88 \text{ kg}$, shown in figure are free to move. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath $M$ is friction-less. What is the minimum horizontal force $F$ required to hold $m$ against $M$?

**Answer:**
The free-body diagrams for both $m$ and $M$ are shown to the right. When $F = \vec{F}_{\text{min}}$

$$f_s = f_{s,\text{max}} = \mu_s N,$$

where $N$ is the normal force exerted by $m$ on $M$. The equations of motion for blocks $m$ and $M$ are

$$\vec{F}_{\text{min}} + \vec{f}_s + mg + \vec{N} = m\vec{a}$$
and

$$\vec{N} = M\vec{a}$$
respectively.

The vector equation above for block $m$ in component form is

$$\begin{cases} F_{\text{min}} - N = ma \\ f_s = \mu_s N = mg; \end{cases}$$

while the equation for $M$ can be written as $N = Ma$. After solving, we obtain the minimum force $F_{\text{min}}$

$$F_{\text{min}} = \frac{mg}{\mu_s} \left(1 + \frac{m}{M}\right) = \frac{16 \times 9.8}{0.38} \left(1 + \frac{16}{88}\right) = 4.9 \times 10^2 \text{ N}.$$

**EXAMPLE 3.8:**
Two blocks, stacked one on the other, slide on a frictionless, horizontal surface. The surface between the two blocks is rough, however, with a coefficient of static friction equal to 0.47.

(a) If a horizontal force $F$ is applied to the 5.0-kg bottom block ($m_2$), what is the maximum value $F$ can have before the 2.0-kg top block ($m_1$) begins to slip?
(b) If the mass of the top block is increased, does the maximum value of $F$ increase, decrease, or stay the same? Explain.

**Answer:**

The maximum frictional force that the 2.0-kg block can experience is $f_{\text{max}}$, where $f_{\text{max}} = \mu m_1 g$, that is $f_{\text{max}} = (0.47)(2.0 \text{ kg})(9.8 \text{ ms}^{-2}) = 9.21 \text{ N}$.

The acceleration of the 2-kg block, $a = f_{\text{max}} / m_1 = 4.61 \text{ ms}^{-2}$.

There will be no slipping when the two block move as one. That is the 5-kg block moves with the same acceleration 4.606 ms$^{-2}$.

According to Newton’s second law, we have $F - f_{\text{max}} = m_2 a$, hence $F = f_{\text{max}} + m_2 a = 9.21 \text{ N} + (5.0 \text{ kg})(4.61 \text{ ms}^{-2}) = 32.3 \text{ N}$.

We observe that the horizontal force $F = \mu m_1 g + \mu m_2 g = \mu g (m_1 + m_2)$. The horizontal force $F$ increases with the mass of the top block.

**EXAMPLE 3.9:**

A 7.96 kg block rests on a plane inclined at 22° to the horizontal, as shown in figure. The coefficient of static friction is 0.25, while the coefficient of kinetic friction is 0.15. (a) What is the minimum force $F$, parallel to the plane, which can prevent the block from slipping down the plane? (b) What is the minimum force $F$ that will start the block moving up the plane? (c) What force $F$ is required to move the block up the plane at constant velocity?

**Answer:**

a) When $F$ is at its minimum, $f_s$ is directed up-hill and assumes its maximum value:

\[ f_{s,\text{max}} = \mu_s N = \mu_s mg \cos \theta. \]

Thus for the block $mg \sin \theta - F_{\text{min}} - \mu_s mg \cos \theta = 0$, which gives

\[ F_{\text{min}} = mg(\sin \theta - \mu_s \cos \theta) \]

\[ = (7.96 \times 9.8)(\sin 22^\circ - 0.25 \times \cos 22^\circ) \]

\[ = 11.14 \text{ N} \]
b) To start moving the block up the plane, we must have $F - mg \sin \theta - \mu_s mg \cos \theta \geq 0$, which gives the minimum value of $F$ required:

$$F_{\text{min}} = mg (\sin \theta + \mu_s \cos \theta)$$

$$= (7.96 \times 9.8)(\sin 22^\circ + 0.25 \times \cos 22^\circ)$$

$$= 47.3 \text{ N}$$

c) In this case the block is already in motion, so we should replace $\mu_s$ with $\mu_k$ in calculation the frictional force. Thus $F - mg \sin \theta - \mu_k mg \cos \theta = ma = 0$, which gives the value of $F$:

$$F = mg (\sin \theta + \mu_k \cos \theta) = (7.96 \times 9.8)(\sin 22^\circ + 0.15 \times \cos 22^\circ)$$

$$= 40.1 \text{ N}$$

### 3.4 Equilibrium Applications of Newton’s Law of Motion

An object either at rest or moving with a constant velocity is said to be in equilibrium. The net force acting on the object is zero. Easier to work with the equation in terms of its components:

$$\sum F_x = 0 \quad \sum F_y = 0$$

#### EXAMPLE 3.10:

A block of mass $m = 15 \text{ kg}$ hanging from three cords, find the tension in three cords.

**Answer:**

Equilibrium condition at knot: $\Sigma F = 0$ (two eqs.)

$x$-direction $F_{Bx} + F_{Ax} = 0$

or $F_B \cos 47 - F_A \cos 28 = 0$ \hspace{1cm} (1)

$y$-direction $F_{Ay} + F_{By} + F_{Cy} = 0$

or $F_B \sin 47 + F_A \sin 28 - mg = 0$ \hspace{1cm} (2)

From Eqs. (1) and (2), we obtain $F_A = 104 \text{ N}$, $F_B = 135 \text{ N}$

#### 3.5 Non-Equilibrium Applications of Newton’s Law of Motion

Similar to equilibrium except

$$\sum F = ma$$
Use components
\[ \sum F_x = m a_x \quad \sum F_y = m a_y \]
a_x or a_y may be zero

**EXAMPLE 3.11:**
Two blocks are connected by a string as shown in the figure. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of \( m_1 = 6.7 \) kg. Find the mass of the hanging block \( m_2 \) that will cause the system to be in equilibrium.

**Answer:**
The free-body diagrams of the blocks \( m_1 \) and \( m_2 \) are shown as follows.

The down-plane component of the 6.7-kg mass is given by
\[ m_1 g \sin 42° = (6.7 \text{ kg})(9.8 \text{ m/s}^2) \sin 42° = 43.9 \text{ N} \]
For equilibrium, this force is balanced if the tension has the same magnitude. On the other hand, the hanging block is also in equilibrium, the weight of it, \( m_2 g \), must balance the tension force, hence we can write
\[ m_2 g = 43.9. \]
The mass of the hanging block is solved as \((43.9 \text{ N})/g = 4.48 \text{ kg} \).
EXAMPLE 3.12:
Find the acceleration and the tension of the system.

Answer:
(a) Free body diagram

(b) About the pulley
If \( m_1 \neq 0 \) and the pulley is rotating, then \( T_2 \neq T_1 \).
In our problem, the pulley is massless and frictionless i.e. \( m_1 = 0 \), hence \( T_1 = T_2 = T \).
The string slides on the pulley.

(c) Cord length is fixed (no extension in the string)
\[ a_1 = a_2 = a \]

(d) Using Newton’s second law
\[
\begin{align*}
T &= Ma \
mg - T &= ma \\
(1) + (2) \quad mg &= (M + m)a \\
\therefore \quad a &= \frac{m}{M + m} \quad g \\
T &= \frac{Mmg}{M + m}
\end{align*}
\]
EXAMPLE 3.13:
Someone exerts a force $F$ directly up on the axle of the pulley shown in figure. Consider the pulley and string to be massless and the bearing frictionless. Two objects, $m_1$ of mass 1.2 kg and $m_2$ of mass 1.9 kg, are attached as shown to the opposite ends of the string, which passes over the pulley. The object $m_2$ is in contact with the floor. (a) What is the largest value the force $F$ may have so that $m_2$ will remain at rest on the floor? (b) What is the tension in the string if the upward force $F$ is 110 N? (c) With the tension determined in part (b), what is the acceleration of $m_1$?

Answer:

a) When the tension of the string greater than the weight of $m_2$, $m_2$ leaves the floor. Therefore the largest value of force $F$ for $m_2$ remain at rest on the floor is

$$F = 2 \times 1.9 \times 9.8$$

$= 37.24$ N

b) Mass of the pulley = 0 kg and net force acts on the pulley = $110 - 2T$

$\therefore 110 - 2T = 0$

$T = 55$ N

c) Net force acts on $m_1 = 55 - 1.2 \times 9.8 = 43.24$ N

Acceleration of $m_1 = \frac{43.24}{1.2} = 36.0$ m/s$^2$
WORK, ENERGY AND POWER

3.1: WORK DONE BY A CONSTANT FORCE

- Work is done by a force when the force moves a body

![Diagram](image)

**Fig. 3.1**

- Fig 3.1 shows a constant force $F$ displacing a body through a small displacement, $s$.
- **DEFINITION**: The work done on an object by a constant force $F$ is $W = (F \cos \theta)s$ ----3.1
  where $F$ is magnitude of the force, $s$ is the magnitude of the displacement, and $\theta$ is the angle between the force and the displacement. Unit: Nm@Joule
- Work can be either positive or negative, depending on whether a component of the force points in the same direction as the displacement or in the opposite direction.

![Diagram](image)

*Work is done when a force $F$ pushes a car through a displacement $s$*
EXAMPLE 3.1 Pulling a suitcase on wheels

(a) Work can be done by a force $F$ that points at an angle $\theta$.

(b) The force component that points along the displacement is $F \cos \theta$.

EXAMPLE 3.2

A force of 50N is used to pull a crate at $30^0$ from horizontal on a smooth floor. If the distance is 3.0m, calculate the work done by the force.

Solution:

$$W = (50 \text{N}) (50 \text{N} \cos 30^0)(3.0 \text{m}) = 130 \text{J}.$$

EXAMPLE 3.3

Calculate work done from force versus displacement graph

The work done by a variable force of magnitude $F$ in moving an object through a displacement of magnitude $s$ is equal to the area under the graph of $F \cos \theta$ versus $s$. The angle $\theta$ is the angle between the force and displacement vectors.

$$\left[ \text{Area under graph from 0 to 3.0 m} \right] = (31 \text{ N} - 0 \text{ N})(3.0 \text{ m} - 0 \text{ m}) = 93 \text{ J}.$$
EXAMPLE 3.4:
Two locomotives are used to pull a ship using a cable with T = 5000 N. Find the total work done by the locomotives if θ is given as 20°.

Each locomotive does work
\[ W = Ts \cos \theta = (5.00 \times 10^3 \text{ N})(2.00 \times 10^3 \text{ m}) \cos 20.0° = 9.40 \times 10^6 \text{ J} \]
J. The net work is then, \[ WT = 2W = 1.88 \times 10^7 \text{ J} \]

3.2: KINETIC ENERGY AND POTENTIAL ENERGY

3.2.1: KINETIC ENERGY AND WORK-ENERGY THEOREM

The kinetic energy KE of an object of mass m and speed v is:

\[ KE = \frac{1}{2}mv^2 \] ----3.2

The relationship that relates work to the change in kinetic energy is known as work-energy theorem. The work-energy theorem states that the work W done by the net external force acting on an object equals the difference between the object’s final kinetic energy KE_f and initial kinetic energy KE_0:

\[ W = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \] ----3.3

The kinetic energy increases when the net force does positive work and decreases when the net force does negative work

- Kinetic energy – is the energy of motion
- Work - positive work (work done on) and negative work (work done by)
- In general, work is a measure of energy transfer, and energy is the capacity of doing work.
Figure: A constant net external force $\Sigma F$ acts over a displacement $s$ and does work on the plane. As a result of a work done, the plane’s kinetic energy changes.

The net force is the vector sum of all the external forces acting on the plane, and it is assumed to have the same direction as the displacement $s$.

$\rightarrow$ Newton’s 2nd law: $\Sigma F = ma$

$\rightarrow$ Multiplying both sides by the distance $s$ gives: $(\Sigma F)s = mas$

$\rightarrow$ The term $as$ on the right side can be related to $v_0$ and $v_f$ by using equation $v_f^2 = v_0^2 + 2as$. By substitute this equation into $(\Sigma F)s = mas$,

$$(\Sigma F)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

**EXAMPLE 3.5**

How much work must be done to stop a 1250kg car traveling at 105km/h?

**Solutions**

The work done on the car is equal to the change in its kinetic energy, and so

$$W = KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = 0 - \frac{1}{2}(1250 \text{ kg}) \left(105 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)\right)^2 = -5.32 \times 10^5 \text{ J}$$

**EXAMPLE 3.7**

A 285kg load is lifted 22.0m vertically with an acceleration $a = 0.160g$ by a single cable. Determine

i. the tension in the cable
ii. the net work done on the load
iii. the work done by the cable on the load.
iv. the work done by the gravity on the load, and e) the final speed of the load assuming it started from rest.

**Solutions:**

(a) From the free-body diagram for the load being lifted, write Newton’s 2nd law for
the vertical direction, with up being positive.

\[ \sum F = F_T - mg = ma = 0.160 mg \]

\[ F_T = 1.16 mg = 1.16 (285 \text{ kg})(9.80 \text{ m/s}^2) = 3.24 \times 10^3 \text{ N} \]

\( \text{(b)} \) The net work done on the load is found from the net force.

\[ W_{\text{net}} = F_{\text{net}}d \cos 0^\circ = (0.160 mg)d = 0.160(285 \text{ kg})(9.80 \text{ m/s}^2)(22.0 \text{ m}) \]

\[ = 9.83 \times 10^3 \text{ J} \]

\( \text{(c)} \) The work done by the cable on the load is

\[ W_{\text{cable}} = F_1d \cos 0^\circ = (1.160 mg)d = 1.16(285 \text{ kg})(9.80 \text{ m/s}^2)(22.0 \text{ m}) = 7.13 \times 10^4 \text{ J} \]

\( \text{(d)} \) The work done by gravity on the load is

\[ W_G = mgd \cos 180^\circ = -mgd = -(285 \text{ kg})(9.80 \text{ m/s}^2)(22.0 \text{ m}) = -6.14 \times 10^4 \text{ J} \]

\( \text{(e)} \) Use the work-energy theory to find the final speed, with an initial speed of 0.

\[ W_{\text{net}} = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \]

\[ v_2 = \sqrt{\frac{2W_{\text{net}}}{m} + v_1^2} = \sqrt{\frac{2(9.83 \times 10^3 \text{ J})}{285 \text{ kg}} + 0} = 8.31 \text{ m/s} \]

**EXAMPLE 3.8**

A 58-kg skier is coasting down a 25° slope as shown below. A kinetic frictional force \( f_k = 70 \text{ N} \) opposes her motion. Near the top of the slope, the skier’s speed is \( v_0 = 3.6 \text{ m/s} \). Ignoring air resistance, determine the speed \( v_f \) at a point that is displaced 57 m downhill.

\[ \text{(b) Free-body diagram for the skier} \]

**SOLUTION**

The external force points along the x axis and is

\[ \Sigma F = mg \sin 25^\circ - f_k = (58 \text{ kg})(9.8 m/s^2)(\sin 25^\circ) - 70 \text{ N} = +170 \text{ N} \]
The work done by the net force is
\[ W = (\Sigma F \cos \theta)s = [170N \cos 0^\circ](57m) = 9700J \]

From work-energy theorem
\[ KE_f = W + KE_0 = 9700J + \frac{1}{2}(58kg)(3.6m/s^2) = 10100J \]
So
\[ v_f = \sqrt{\frac{2(KE_f)}{m}} = \sqrt{\frac{2(10100J)}{58kg}} = 19m/s \]

### 3.2.2: POTENTIAL ENERGY

**WORK DONE BY THE FORCE OF GRAVITY**

The gravitational potential energy of a body is due to its relative position in a gravitational field.

The drawing depicts a basketball of mass m moving vertically downward, the force of gravity mg being the only force acting on the ball. The ball displacement s is downward and has a magnitude of
\[ s = h_0 - h_f \]

To calculate the work, \( W_{\text{gravity}} \) done on the ball by the force of gravity, we use
\[ W = (F \cos \theta)s \]
With \( F = mg \) and \( \theta = 0^\circ \)

\[ W_{\text{gravity}} = (mg \cos 0^\circ)(h_0 - h_f) = mg(h_0 - h_f) \]

Equation 3.4 valid for any path taken between the initial and final heights, and not just for the straight down path.
An object can move along different paths in going from an initial height \( h_0 \) to a final height of \( h_f \). In each case, the work done by the gravitational force is the same [ \( W_{\text{gravity}} = mg(h_0 - h_f) \)], since the change in vertical distance \( (h_0-h_f) \) is the same.

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**GRAVITATIONAL POTENTIAL ENERGY**

To lift a mass \( m \) at a constant velocity, the force required is

\[ F = mg \]

When the mass is displaced through a height \( h \),

\[ W = Fs = mgh \]

Definition: The energy that an object of mass \( m \) has by virtue of its position relative to the surface of the earth. That position is measured by the height \( h \) of the object relative to an arbitrary zero level;

\[ PE = mgh \]

The work done increases the gravitational potential energy of the mass. Hence, when a mass \( m \) is raised through a height \( h \), increase in gravitational potential energy is \( mgh \). When a mass \( m \) drops through a vertical distance of \( h \), the gravitational potential energy decreases by \( mgh \).
From figure above, the pile driver contains a massive hammer that is raised to a height \( h \) and then dropped. As a result, the hammer has the potential to do the work of driving the pile into the ground. The greater the height of the hammer, the greater is the potential for doing work, and the greater is the gravitational potential energy.

**EXAMPLE 3.9**

By how much does the gravitational potential energy of a 64 kg high jumper change if his center of mass (body) rises about 4.0m during the jump?

**Solutions:**

\[
\Delta \text{PE}_{\text{grav}} = mg(y_2 - y_1) = (64 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) = 2.5 \times 10^3 \text{ J}
\]

**EXAMPLE 3.10**

A 1.60m tall person lifts a 2.10kg book from the ground so it is 2.20m above the ground. What is the potential energy of the book relative to a) the ground, and b) the top of the person’s head? C) How is the work done by the person related to the answers in parts a) and b)?

**Solutions:**

\( (a) \) Relative to the ground, the PE is given by

\[
P \text{E}_{\text{gi}} = mg \left( y_{\text{book}} - y_{\text{ground}} \right) = (2.10 \text{ kg})(9.8 \text{ m/s}^2)(2.20 \text{ m}) = 45.3 \text{ J}
\]

\( (b) \) Relative to the top of the person’s head, the PE is given by

\[
P \text{E}_{\text{gi}} = mg \left( y_{\text{book}} - y_{\text{head}} \right) h = (2.10 \text{ kg})(9.8 \text{ m/s}^2)(0.60 \text{ m}) = 12 \text{ J}
\]

\( (c) \) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part \( (a) \), \( 45.3 \text{ J} \). In part \( (a) \), the PE is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part \( (b) \).
ELASTIC POTENTIAL ENERGY

![Diagram of elastic potential energy](image)

(Giancoli 6th ed. Pg 147)

Elastic potential energy:

\[ PE = \frac{1}{2} kx^2 \]

**EXAMPLE 3.11**

A 1200kg car rolling on a horizontal surface has speed \( v = 65 \text{km/h} \) when it strikes a horizontal coiled spring and is brought to rest in a distance of 2.2m. What is the constant of the spring?

**Solutions:**

Assume that all of the kinetic energy of the car becomes PE of the compressed spring.

\[
\frac{1}{2} mv^2 = \frac{1}{2} kx^2 \quad \Rightarrow \quad k = \frac{mv^2}{x^2} = \frac{(1200 \text{ kg})\left(65 \text{ km/h}\right)\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)^2}{(2.2 \text{ m})^2} = 8.1 \times 10^4 \text{ N/m}
\]

3.3: THE CONSERVATION OF ENERGY

The principle of conservation of energy is energy can either be created or destroyed, but only can be converted from one form to another. Whenever energy is transformed from one form to another, it is found that no energy is gained or lost in the process, the total of all the energies before the process is equal to the total of the energies after process.
**Conservative versus nonconservative forces**

Definition of conservative force
1. A force is conservative when the work it does on a moving object is independent of the path between the object’s initial and final positions.
2. A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point.
   
   Example: Gravitational force, elastic spring force and electric force

![Roller coaster track](image)

Figure: A roller coaster track is an example of a closed path.

Definition of nonconservative force
1. A force is nonconservative if the work it does on an object moving between 2 points depends on the path of the motion between the points
2. For a closed path, the total work done by a nonconservative force is not zero
   
   Example: Friction force, air resistance, tension, normal force, propulsion force of a rocket

Work done by the net external force;

\[ W = W_c + W_{nc} \]

According to the work-energy theorem, the work done by the external force is equal to the change in the object’s kinetic energy;

\[ W_c + W_{nc} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

If the only conservative force acting is the gravitational force, then

\[ W_c = W_{\text{gravity}} = mg(h_0 - h_f) \]

Hence

\[ W_{nc} = \left( \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \right) + \left( mgh_f - mgh_0 \right) \quad ----3.6 \]

In terms of kinetic and potential energies, we find

\[ W_{nc} = (KE_f - KE_0) + (PE_f - PE_0) = \Delta KE + \Delta PE \quad ----3.7 \]
3.4: THE CONSERVATION OF MECHANICAL ENERGY (WORK-ENERGY THEOREM)

The total mechanical energy $E$ is formed by combining the concepts of kinetic energy and gravitational energy.

The principle of conservation of mechanical energy is the total mechanical energy ($E=KE+PE$) of an object remains constant as the object moves, provided that the net work done by external nonconservative forces is zero.

The work energy theorem can be expressed in terms of the total mechanical energy:

\[
W_{nc} = (KE_f - KE_0) + (PE_f - PE_0)
\]

\[
W_{nc} = (KE_f + PE_f) - (KE_0 + PE_0)
\]

\[
E_f = E_0
\]

Suppose that the net work $W$ done by external nonconservative forces is zero, so $W=0$. So,

\[
\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0
\]

Figure above shows the transformations of energy for a bobsled run, assuming that nonconservative forces, such as friction and wind resistance, can be ignored. Kinetic and potential energy can be in converted, while the total mechanical energy remains constant.

**EXAMPLE 3.12**
A novice skier, starting from rest, slides down a frictionless $35^0$ incline whose vertical height is 185m. How fast is she going when she reaches the bottom?

**Solutions:**
The forces on the skier are gravity and the normal force. The normal force is perpendicular to the
direction of motion, and so does no work. Thus the skier’s mechanical energy is conserved.
Subscript 1 represents the skier at the top of the hill, and subscript 2 represents the skier at
the bottom of the hill. The ground is the zero location for PE \( y = 0 \). We have \( v_1 = 0 \), \( y_1 = 185 \text{ m} \),
and \( y_2 = 0 \) (bottom of the hill). Solve for \( v_2 \), the speed at the bottom.

\[
\frac{1}{2} mv_1^2 + mgy_1 = \frac{1}{2} mv_2^2 + mgy_2 \rightarrow 0 + mgy_1 = \frac{1}{2} mv_2^2 + 0 \rightarrow \\
v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(185 \text{ m})} = 60.2 \text{ m/s} \approx 135 \text{ mi/h} 
\]

3.5: NONCONSERVATIVE FORCES AND THE WORK ENERGY THEOREM

In this case, the final and initial total energies is equal to \( W \);

\[
W = E_f - E_0
\]

EXAMPLE 3.13
A 145g baseball is dropped from a tree 13.0 m above the ground.

a) With what speed will it hit the ground if air resistance could be ignored?
b) If it actually hits the ground with a speed of 8.0m/s, what is the average force of air
resistance exerted on it?

Solutions:

(a) Apply energy conservation with no non-conservative work. Subscript 1 represents the ball as it
is dropped, and subscript 2 represents the ball as it reaches the ground. The ground is the zero
location for gravitational PE. We have \( v_1 = 0 \), \( y_1 = 13.0 \text{ m} \), and \( y_2 = 0 \). Solve for \( v_2 \).

\[
E_1 = E_2 \rightarrow \frac{1}{2} mv_1^2 + mgy_1 = \frac{1}{2} mv_2^2 + mgy_2 \rightarrow mgy_1 = \frac{1}{2} mv_2^2 \rightarrow \\
v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(13.0 \text{ m})} = 16.0 \text{ m/s}
\]

(b) Apply energy conservation, but with non-conservative work due to friction included. The work
done by friction will be given by \( W_{nc} = F_n d \cos 180^\circ \), since the force of friction is in the
opposite direction as the motion. The distance \( d \) over which the frictional force acts will be the
13.0 m distance of fall. With the same parameters as above, and \( v_2 = 8.00 \text{ m/s} \), solve for the
force of friction.

\[
W_{nc} + E_1 = E_2 \rightarrow -F_n d + \frac{1}{2} mv_1^2 + mgy_1 = \frac{1}{2} mv_2^2 + mgy_2 \rightarrow -F_n d + mgy_1 = \frac{1}{2} mv_2^2 \rightarrow \\
F_n = m \left( g \frac{y_1 - v_2^2}{2d} \right) = (0.145 \text{ kg}) \left( 9.80 \text{ m/s}^2 - \frac{(8.00 \text{ m/s})^2}{2(13.0 \text{ m})} \right) = 1.06 \text{ N}
\]
3.6 POWER

Average power, $P$ is defined as the average rate of doing work

$$ P = \frac{\text{work}}{\text{time}} = \frac{W}{t} \quad \text{-------} 3.10 $$

$$ \frac{W}{t} = \frac{F.s}{t} $$

So,

$$ P = Fv \quad (v \text{ is speed}) \quad \text{-------} 3.11 $$

For example, when a machine does more work in a shorter time, the power of the machine is higher.

SI unit $\rightarrow$ watt ($W$) @ joule per second

$1kW = 1000W$

Energy or work done sometimes also measured in kilowatt-hour (kWh)

$1kWh = (1x10^3)(60x60)J = 3.6\times10^3 J$

**EXAMPLE**

A 1000kg elevator carries a maximum load of 800kg. A Constant frictional force of 4000N retards its motion upward. What minimum power must the motor deliver to lift the fully loaded elevator at a constant speed of 3.0m/s?

**Solution:**

$$ T - f = Mg $$

Where $M$ is the total mass = 1800 kg. Therefore,

$$ T = f + Mg = 4000N + (1800\text{kg})(9.81\text{m/s}^2). $$

$$ = 2.16 \times 10^4 \text{ N}. $$

using $P = F \bar{V} \quad , \quad P = T \bar{V}$

$$ P = 2.16 \times 10^4 \text{ N} \times 3.0 \text{ m/s} = 6.48 \times 10^4 \text{ W}$$

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